

# WHAT HAPPENS WHEN YOU REGULATE RISK? EVIDENCE FROM A SIMPLE EQUILIBRIUM MODEL\*

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## Abstract

Global financial regulations are increasingly becoming risk sensitive, with Value-at-Risk a key component. The economic implications of a Value-at-Risk based regulatory system are analyzed in a multi-asset general equilibrium model with agents that are heterogeneous in risk preferences, wealth, and degree of supervision. Excessive leverage and risk taking arise due to externalities, giving rise to systemic risk. The model suggests that risk sensitive regulation can lower systemic risk, at the expense of poor risk-sharing, an increase in risk premia, higher asset volatility, lower liquidity, more comovement in prices, and the chance that markets may not clear. However, systemic risk can be worsened by risk sensitive regulation if too many institutions are left out of the regulatory umbrella.

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# 1 Introduction

Risk sensitive regulation, where statistical risk models are used to determine allowable levels of risk and of bank capital, has recently become the cornerstone of international financial regulations. The equilibrium and welfare properties of such regulatory methodology have proven difficult to analyze, especially in view of the lack of formal models of systemic risk, and have received scant attention in the economics literature. Our objective is to formally analyze both the possible economic justifications of risk sensitive regulation, as well as their economic impact. We discuss some possible externalities mitigated by such regulation, and show circumstances in which systemic risk can be reduced. We also identify several perverse outcomes of the actual regulatory mechanism. In particular, we argue that some of the costs of risk sensitive regulation consist of suboptimal risk-sharing, the possibility markets can't clear as well as an *increase* in systemic risk in some circumstances. Value-at-Risk based regulation also endogenously changes the statistical properties of prices and returns in financial markets.

Traditional forms of bank regulation have mostly taken the form of lending of last resort, deposit insurance, and activity restrictions (see e.g. Bagehot, 1873; Diamond and Dybvig, 1983; Allen and Gale, 1998). In general, such regulation is on a national or local level, involving limited international coordination. Subsequent to the fall of Bankhaus Herstatt in 1974 and Banco Ambrosiano in 1982, the Bank of International Settlements (BIS) and the Basel Committee (BC) received a mandate to propose risk sensitive banking regulations for the largest economies. First to be introduced was the 1988 Capital Adequacy Accord which was based on using crude risk weights to determine bank capital for credit exposures. In a 1996 amendment, market risk was added as a determinant of capital, where in a significant development, market risk capital is determined by a mathematical formula which is a function of a statistical risk measure, Value-at-Risk (VaR).<sup>1</sup> This is known as the internal ratings approach (IRB). The next iteration of this process (Basel-II) extends the IRB methodology to both credit and operational risk. Embedded in these approaches to banking regulation is the view held by banking supervisors that market prices are exogenous. Under the Basel agreements, bank capital is explicitly a function of a statistical risk measure, which is to be estimated with historical market prices. Therefore, the potential of economic agents for altering their behavior in the presence of regulation is implicitly disregarded, since the severity of the constraints changes with market conditions, where the constraints are especially binding in market crashes.

Our benchmark model (also termed the unregulated economy) is a generalization of the standard workhorse in financial economics. There are two periods. In the first period agents receive (possibly heterogeneous) endowments of a number of risky assets as well

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<sup>1</sup>In particular,  $\text{capital} = 3 \times \text{VaR} + \text{constant}$  where Value-at-Risk is given by the solution to  $0.01 = \int_{-\infty}^{\text{VaR}} \widehat{f}(x) dx$ , where  $\widehat{f}(x)$  is the estimated probability density function of a financial institutions profit and loss.

as a riskless asset. The risky assets provide normally distributed payoffs in the second period. The agents are heterogeneous in risk aversion, and have CARA utility. CARA utility allows for closed-form solutions, and we argue that the inherent logic of our results would carry over to more general specifications. There are also noise traders who submit market orders. Equivalently, investors' asset endowments are random. This in turn induces trading, price formation, and a price volatility level. No assumptions are made as to the distribution of the noise trades or the initial endowments, and their sole role is to introduce price randomness. In a dynamic model with gradual information revelation from dividends, the noise traders or the random asset endowments could be dispensed with. This economy, in which market clearing is assured, provides the benchmark with which the regulated economy is compared. The benchmark economy results in first-best outcomes and hence has no externalities that merit regulation.

In the real economy the notion of *systemic crises* is the *raison d'être* for financial regulation. Unfortunately no single definition of systemic risk exists (see De Bandt and Hartmann, 2000, for a survey). Some understanding of the underlying motivation for risk sensitive regulations can be obtained by studying public statements made by public officials. Andrew Crockett (2000) states that one objective is "limiting the costs to the economy from financial distress" where the seeds of financial crises are sown by "excessive risk taking."

The task of incorporating such political objectives into our model is challenging due to the lack of micro-based models that can be readily adapted. While the models of Allen and Gale (2000) and Diamond and Rajan (2001) may perhaps be the closest candidates in the literature, the mechanisms of a systemic crisis and the ensuing liquidity shortage we have in mind are more market and less banking based. We consider a systemic crisis to be an event where the collapse of the credit system causes the collapse of the real productive sector. While many papers have discussed systemic risk, we are not aware of any that have actually modelled the effects of risk-regulation (as opposed to lender-of-last-resort mechanisms in bank-run models for instance) upon systemic risk. In our paper, a free-riding externality induces agents to disregard the social cost of their actions on the global system. The probability of a systemic crash increases along with imbalances in agents' leverage and risk taking. We measure systemic risk by the degree of imbalance of risk taking and leverage among agents.

In theory, this formulation of systemic crises would suggest some form of optimal regulation. However, the objective of this paper is to analyze existing regulatory structures, and not to propose new forms of regulation. We therefore adopt the general framework of the 1996 Amendment and the Basel-II proposals (Value-at-Risk), and limit the amount of risk agents are allowed to bear. Providing that these risk constraints are sufficiently restrictive and inclusive, such regulation can effectively reduce or even eliminate systemic risk. However, since this represents a second best solution, the benefits should be counter-balanced against the potential side effects. Imposing Basel style constraints on the benchmark economy obviously has real consequences on outcomes (also refer to Basak and Shapiro (2001)

for a continuous-time frictionless representative-agent model with log utility and complete markets in which a VaR constraint is imposed upon the investor), and a detailed understanding of these secondary impacts is essential for the effective evaluation of the pros and cons of the chosen regulatory structure. In the risk-constrained economy risk-sharing, risk premia, volatility, liquidity, asset price comovement, and market clearing are all affected. Furthermore, the tighter the constraint, the greater the impact becomes.

In our first main result we demonstrate that compared to the benchmark economy, optimal risk-sharing will be impeded, liquidity will be lower in the regulated economy, whilst volatility and risk premia will be higher (which we argue might address part of the equity-premium puzzle). With the formal results presented in Propositions 3, 4 and 6, for easy visualization, consider the single risky-asset economy in Figure 3 where we depict the equilibrium price of the risky asset as a function of the noise trades. The regulatory constraint causes the pricing function to become more concave for typical trades, since risk will have to be transferred from the more risk-tolerant to the more risk-averse. In order for the more risk-averse to take on the additional risk, the discount will have to be bigger the more risk-averse the marginal buyers are. The slope, which represents the market impact of a trade, therefore increases with the severity of regulation. Hence for a given change in demand, prices move more with regulation than without, implying higher (local) volatility and lower liquidity post regulation. In a crisis, we expect financial institutions to have to sell assets, e.g. to meet margin and regulatory requirements. However, since in that case we move to the left in the figure, liquidity diminishes and (local) volatility shoots up.

It is well known in the empirical literature that correlations or comovements of assets are amplified in times of stress. While margin calls and wealth effects have been among the proposed explanations, as in Kyle and Xiong (2001), we are not aware of any models that are able to generate increased comovements in periods of stress from the regulatory constraints. Our model suggests that one further explanation for the observed state-dependent comovement may be the impact of risk constraints on portfolio optimization, especially in times of stress. Even if assets have independent payoffs, sufficiently binding regulations will cause some agents to adjust their risk position by scaling down their holdings in the risky assets, thereby introducing comovements. This effect will be most pronounced during financial crisis. As a result, a Basel style regulation introduces the potential for an endogenous increase in correlation, thereby decreasing the agents ability to diversify and increasing the severity of financial crises. Financial institutions therefore require higher risk premia, which may address at least part of the equity-premium puzzle.

Our model also demonstrates that regulatory risk constraints may prevent market clearing in some circumstances if all financial institutions are in fact regulated. The probability of markets not clearing increases with the tightness of the risk constraint. The basic intuition can be seen in the two asset economy in Figure 1. In the absence of regulation, equilibrium is assured for all possible liquidity sales, but as risk constraints become increasingly binding, the equilibrium can be supported only for liquidity sales within a shrinking ellipsoid. For

trades outside the ellipsoid, the risk that would have to be taken on by the buyers must violate at least some buyers' regulatory constraints. We term the absence of market clearing as *market breakdown*. The only way to ensure market clearing in all circumstances is to exempt some institutions from risk constraints.

Our results indicate that the specific form of risk sensitive regulation can have a considerable impact on the financial sector. Within our framework, sufficiently restrictive and inclusive risk regulation does reduce the probability of a systemic crisis. VaR regulation may, however, hasten a systemic event if sufficiently many institutions escape the regulatory umbrella. Either way, risk regulation impedes the exploitation of gains from trade and from risk-sharing. To conclude, the potential of the regulations endogenously changing the statistical properties of market variables does not appear to be adequately recognized by banking supervisors. Analysis of documents and statements issued by the Basel Committee (see [www.bis.org](http://www.bis.org)) suggests that these bodies consider market prices to be exogenous, and that risk sensitive regulation will not change the statistical properties of prices. In particular, the notion that risk sensitive regulation may be procyclical is largely disregarded.

The structure of the paper is as follows. In Section 2 we present the ingredients of the model. Section 3 discusses a financial institution's decision problem, and Section 4 establishes the necessary and sufficient conditions for the existence of equilibria. In Section 5 we study the impact on risk-taking, depth and volatility, and we analyze financial crises in Section 6. Section 7 concludes. Most proofs are contained in the Appendix, and all figures are at the end of the paper.

## 2 The Model

Our economy is a standard two period constant absolute risk–aversion model without asymmetric information and with stochastic asset supply. There are three families of agents: regulated financial institutions (RFI) that are subjected to regulatory risk constraints (e.g. banks), unregulated institutions (UFI) (e.g. hedge funds), and noise traders (equivalently, random asset endowments). In the first period the UFIs and RFIs invest their endowments in both risky and riskless assets, and the noise traders submit an aggregate market order for assets. Consumption occurs at the second date. We follow common modelling practice by endowing financial institutions with their own utility functions (such as in Basak and Shapiro, 2001, for instance). While delegated portfolio management is certainly an important issue, we have no reason to believe that the logic as to how VaR constraints affect asset prices would be significantly different.<sup>2</sup>

There are  $N$  nonredundant risky assets that promise normally distributed payoffs  $\mathbf{d} \sim N(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\Sigma}})$  at time 1, independent of the noise trader supply of assets. The hats indicate payoffs, returns are distributed  $N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ . Asset 0 is the riskless asset and promises to pay off  $d_0$ .

Each type of financial institution  $h \in [\ell, 1]$  is characterized by a constant coefficient of absolute risk aversion (CARA)  $\alpha^h$  as well as an initial endowment of the riskless asset  $\theta_0^h$  and of the risky assets  $\boldsymbol{\theta}^h$ . They do not have any other state-contingent endowments or preference shocks. For simplicity, we assume that  $\alpha^h = h$  (but for clarity we still label agent  $h$ 's coefficient by  $\alpha^h$  rather than by  $h$  only) and that all institutions are risk-averse,  $\ell > 0$ . A fraction  $\eta$  of agents of each type  $h$  are regulated, the remaining fraction is unregulated.

We do not model the noise traders' utility explicitly, and only assume that they are hit by liquidity shocks at time 0 which cause them to submit an aggregate market order for  $\boldsymbol{\epsilon}$  assets. This demand is assumed to be distributed on  $\mathbf{E} \subset \mathbb{R}^N$  according to the law  $\mathbb{P}^\epsilon$ , for simplicity assumed to be independent of the law governing asset payoffs,  $\mathbb{P}^d$ . In this paper, we do *not impose any* assumptions upon the distribution of  $\boldsymbol{\epsilon}$  other than to assume that its support  $\mathbf{E}$  is open and convex, in order to occasionally apply differential calculus. Because the market order has to be absorbed by the UFIs and RFIs, prices depend upon  $\boldsymbol{\epsilon}$ . This is the only role of noise traders (also see Footnote 6 below). In a dynamic version of our model where dividends or news about the value of firms govern the resolution of uncertainty, they can be dropped entirely, as in Danielsson et al. (2003).

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<sup>2</sup>If FIs were themselves traded, delegated portfolio management issues would of course affect the stock prices of the FIs.

### 3 Decision Problem of the Financial Institutions

Each FI is characterized by its type  $h$ , which determines risk-aversion and endowments, and by its regulation status  $t$ , which is either  $t = \{r\}$  if the FI is regulated, or  $t = \{u\}$  if it is unregulated. A FI  $(h, t)$  invests its initial wealth  $W_0^h$  in a portfolio comprising both riskless and risky assets,  $(y_0^{h,t}, \mathbf{y}^{h,t})$ . The time-zero wealth of an agent of type  $h$  (regulated or unregulated) comprises initial endowments in the riskless asset,  $\theta_0^h$ , as well in risky assets,  $\boldsymbol{\theta}^h$ , so that  $W_0^h \equiv q_0\theta_0^h + \mathbf{q}'\boldsymbol{\theta}^h$ . The price vector of risky assets is denoted by  $\mathbf{q}$ . The aggregate amount of outstanding risky assets is  $\boldsymbol{\theta}^a := \int_{\ell}^1 \boldsymbol{\theta}^h dh$ . Since the time-zero budget constraint  $q_0\theta_0^h + \mathbf{q}'\boldsymbol{\theta}^h \geq q_0y_0^{h,t} + \mathbf{q}'\mathbf{y}^{h,t}$  is homogeneous of degree zero in prices, we can normalize, without loss of generality, the price of the riskless asset to  $q_0 \equiv 1$ , i.e. the riskless asset is used as the time-zero numéraire. We can write  $R_f := d_0/q_0 = d_0$  for the return on the riskless asset. At time 1, the consumption commodity plays the role of the numéraire.

The aim of the regulations for risk-taking is to control extreme risk-taking by individual financial institutions. Here we focus our attention on how risk constraints affect individual financial institutions as well as the entire financial sector, assuming that no systemic event occurs, and return to systemic collapses in Section 6. In theory, a large number of possible regulatory environments exist for this purpose. In practice, we are not aware of any published research into the welfare properties of alternative market risk regulatory methodologies,<sup>3</sup> and as a result, we adopt the standard market risk methodology, i.e., Value-at-Risk. The constraint takes the form (we drop the superscript  $r$  whenever no confusion arises):

$$\mathbb{P}^d [(E^d[W^h] - W^h) \geq VaR] \leq \bar{p},$$

i.e. the probability of a *loss* larger than the uniform regulatory number  $VaR$  is sufficiently small. Since the portfolio payoffs are normal, a sufficient statistic for portfolio risk is the volatility of  $W^h$ . The VaR constraint can therefore be stated as an exogenous upper bound  $\bar{v}$  on portfolio variance,<sup>4</sup>

$$\mathbf{y}^{h'} \hat{\Sigma} \mathbf{y}^h \leq \bar{v}. \quad (1)$$

Each RFI maximizes the expected utility subject to both the budget constraint and the

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<sup>3</sup>Such as various schemes to explicitly limit risk-taking or leverage, versus lending-of-last-resort, regulation of the admissible financial contracting practices with a view of overcoming agency or free-riding problems, and so forth. Of course, we know from Artzner et al. (1999) that VaR is not a desirable measure from a *purely statistical* point of view because it fails to be subadditive. Furthermore, Ahn et al. (1999) show that the VaR measure may not be reliable because it is easy for a financial institution to legitimately manage reported VaR through options.

<sup>4</sup>Indeed, denoting the cumulative standard normal distribution function by  $N(\cdot)$ , the VaR constraint can be reduced to a volatility constraint:  $\mathbb{P}^d [(E^d[W^h] - W^h) \geq VaR] \leq \bar{p}$  iff  $N\left(\frac{-VaR}{\text{Std}^d(W^h)}\right) \leq \bar{p}$  iff  $\text{Std}^d(W^h) \leq \frac{VaR}{-N^{-1}(\bar{p})}$  iff  $\text{Var}^d(W^h) \leq \bar{v} := \left(\frac{VaR}{-N^{-1}(\bar{p})}\right)^2$ .

VaR constraint by choosing the optimal asset holdings. Lemma 1 in the Appendix shows that the optimal portfolio for RFI  $(h, t)$  has the mean-variance form

$$\mathbf{y}^{h,t} = \frac{1}{\alpha^h + \phi^{h,t}} \hat{\Sigma}^{-1} (\hat{\boldsymbol{\mu}} - R_f \mathbf{q}) \quad (2)$$

where  $\phi^{h,u} := 0$  and  $\phi^{h,r} := \frac{2\lambda^{h,r}}{E^d[u^{h'}(W^h)]} \geq 0$ , with  $\lambda^{h,r}$  being the Lagrange multiplier of the VaR constraint. We see that a binding risk-regulation affects the portfolio through the effective degree of risk-aversion,  $\alpha^h + \phi^{h,r}$ . Whereas the coefficient of absolute risk-aversion is constant for unrestricted FIs, it is effectively endogenous for the FIs subjected to the VaR regulations and larger than their utility-based coefficient during volatile events,  $\alpha^h + \phi^{h,r} \geq \alpha^h$ . In volatile events RFIs shift wealth out of risky assets into the safe haven provided by the riskless asset. This is one way of capturing the often-heard expression among practitioners that “risk-aversion went up.” This is reminiscent of the effect of portfolio insurance on optimal asset holdings found in Grossman and Zhou (1996). Also see Basak and Shapiro (1995) and Gennotte and Leland (1990). As a matter of convention, we reserve the term *risk-aversion* to the CARA coefficients  $\alpha^h$ . We call  $\alpha^h + \phi^{h,r}$  the coefficient of *effective risk-aversion*, and we call its inverse *risk appetite*.

Market clearing prices require that the total excess demand by regulated and unregulated institutions,  $\eta \int_{\ell}^1 \mathbf{y}^{h,r} dh + (1 - \eta) \int_{\ell}^1 \mathbf{y}^{h,u} dh - \boldsymbol{\theta}^a$  plus the excess demand by noise traders,  $\boldsymbol{\epsilon}$ , must equal zero. Equivalently they satisfy the relation:

$$\mathbf{q} = \frac{1}{R_f} \left[ \hat{\boldsymbol{\mu}} - \Psi \hat{\Sigma} (\boldsymbol{\theta}^a - \boldsymbol{\epsilon}) \right] \quad (3)$$

where

$$\Psi^{-1} := \eta \int_{\ell}^1 \frac{1}{\alpha^h + \phi^{h,r}} dh + (1 - \eta) \int_{\ell}^1 \frac{1}{\alpha^h} dh \quad (4)$$

is the aggregate effective risk-tolerance.  $\Psi$  can also be viewed as the reward-to-variability ratio (or a market-price of risk scalar) of the residual market  $\boldsymbol{\theta}^a - \boldsymbol{\epsilon}$ ,  $\Psi = \frac{\hat{\boldsymbol{\mu}}_{\text{RM}} - R_f \mathbf{q}_{\text{RM}}}{\hat{\sigma}_{\text{RM}}^2}$ . Compared to an economy without any VaR constraints where the market-price of risk scalar is  $\gamma := \left( \int_{\ell}^1 \frac{1}{\alpha^h} dh \right)^{-1}$ , we have  $\Psi \geq \gamma$ . But the market price of risk is not only higher in a constrained economy than in an unconstrained one, it also is endogenous through the additional risk aversion  $\phi^h$  imposed by the regulations.<sup>5 6</sup>

<sup>5</sup>Equations (2, 3 and 4) remain valid if utility functions are not of the constant absolute risk-aversion class. The only difference would be that  $\alpha^h = \frac{-E^d[u^{h''}]}{E^d[u^{h'}]}$ , and therefore endogenous. While no closed-form solutions exist in this more general case, the rationale why the *aggregate* effective risk-aversion satisfies  $\Psi \geq \gamma$  would still hold with reasonable income effects. Since most results in the sequel are driven mainly by the fact that risk-constraints effectively lower aggregate risk-tolerance, we feel comfortable as to the robustness of the results derived here.

<sup>6</sup>The sole pricing factor being the residual market portfolio  $\boldsymbol{\theta}^a - \boldsymbol{\epsilon}$ , it becomes apparent that assuming



## 4 On Hedge Funds and Market Clearing

Our definition of a competitive equilibrium as a pricing function  $Q$  mapping noise trades  $\epsilon$  to market clearing prices is entirely standard. Proposition 1 solves for the equilibrium  $\Psi$  (see Equation (13) in the Appendix for an exact expression) and prices:

**Proposition 1 (Existence)** *If  $\eta < 1$ , there exists a unique competitive equilibrium for any  $(\epsilon, \bar{v}, \ell) \in \mathbf{E} \times [0, \infty) \times (0, 1]$ .*

*If  $\eta = 1$ , there exists an equilibrium for  $\ell \in [0, 1]$  and for  $(\bar{v}, \epsilon)$  satisfying  $\epsilon \in \mathcal{E}(\bar{v}, \ell) := \{\epsilon \in \mathbf{E} : [(\theta^a - \epsilon)' \hat{\Sigma}(\theta^a - \epsilon)]^{1/2} \leq (1 - \ell)\sqrt{\bar{v}}\}$ . For  $(\bar{v}, \epsilon)$  such that  $\epsilon \in \text{int } \mathcal{E}(\bar{v}, \ell)$ , the equilibrium is unique, while for  $(\bar{v}, \epsilon)$  such that  $\epsilon \in \partial \mathcal{E}(\bar{v}, \ell)$  asset prices and consumption allocations are indeterminate (within a certain range of prices) but the allocation of risky assets is not. No equilibrium exists for  $\epsilon \notin \mathcal{E}(\bar{v}, \ell)$ .*

Equilibria always exist if there are unregulated financial institutions ( $\eta < 1$ ). If all institutions are regulated ( $\eta = 1$ ), then there are combinations of regulatory levels  $\bar{v}$  and noisy asset supplies  $\epsilon$  in which markets cannot clear. This happens precisely if the noise trader supply that has to be absorbed by the regulated financial institutions is such that the per-capita amount of risk that would need to be held to clear markets is higher than the level allowed by regulation  $\bar{v}$ :  $\sqrt{(\theta^a - \epsilon)' \hat{\Sigma}(\theta^a - \epsilon)}/(1 - \ell) > \sqrt{\bar{v}}$ . Alternatively, markets cannot clear if the number of agents over which the risk needs to be evenly spread,  $\kappa(\epsilon; \bar{v}) := \sqrt{\frac{(\theta^a - \epsilon)' \hat{\Sigma}(\theta^a - \epsilon)}{\bar{v}}}$ , is larger than the population:  $\kappa > 1 - \ell$ .

In the present regulatory environment, some FIs, e.g. hedge funds, are exempted from the risk regulations. Following the collapse of LTCM, some policymakers have discussed whether to bring hedge funds under the regulatory umbrella. We can analyze the impact of regulating hedge funds within the model and see how this affects market outcomes. Our starting point is a situation where a fraction  $1 - \eta > 1$  of FIs is unregulated. We know that equilibria exist in this case.

Any large supply of risky assets  $\epsilon \in \mathbf{E}$  has to be absorbed by the FIs. Intuitively, as  $\eta$  is raised further towards 1, more and more FIs are subjected to a maximum level of risk they can take on, and prices adjust to guide the supply towards the institutions whose constraint is not binding yet. But as  $\eta \rightarrow 1$ , there are no such institutions left. So for noise trades sufficiently different from  $\theta^a$ , no price will be able to induce market-clearing. This is not a short-coming of our model. In fact, *any* model would exhibit such a result as it relies solely on the universality of VaR constraints. Figure 1 illustrates this phenomenon in an economy with two assets and different levels of tightness  $\bar{v}$ . Each level of tightness determines an ellipsoid set of noise supplies that can be supported by a competitive equilibrium. For  $\epsilon$

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noise traders is equivalent to assuming a random aggregate endowment in risky assets of  $\theta^a - \epsilon$ , distributed appropriately among investors.

outside of this ellipsoid, FIs cannot absorb the supply as described earlier, and markets break down. And for a tighter regulatory level  $\bar{v}_2 < \bar{v}_1$ , the set of supportable supplies shrinks even further,  $\mathcal{E}(\bar{v}_2) \subset \mathcal{E}(\bar{v}_1)$ .

This suggests the policy implication that if the supervisory authorities impose stringent risk limits (in the sense that  $\bar{v}$  is small enough to lead to  $\mathcal{E}(\bar{v}) \not\supseteq \mathbf{0}$ , i.e.  $\bar{v} < \frac{\boldsymbol{\theta}^a \hat{\boldsymbol{\Sigma}} \boldsymbol{\theta}^a}{(1-\ell)^2}$ ), some agents need to be exempted from those constraints for markets to clear, i.e.  $\eta < 1$  is needed. This observation suggests that demands for the regulation of hedge funds may be misguided, at least for assets that are not in zero net supply. As an illustration, assume in Figure 1 with a regulatory level  $\bar{v}_2$  that noise traders dump assets (an irrational panic, say), leading to  $\boldsymbol{\epsilon} \in \mathbb{R}_-^2$ . No matter how low prices fall, there is no price level low enough for RFIs to be able to absorb this supply since they are all prevented from holding the risk. Hedge funds on the other hand are the natural buyers for undervalued assets, and if given the opportunity to buy into the selling, they may restore equilibrium.

For derivatives, however,  $\mathbf{0} \in \mathbf{E} \subseteq \mathcal{E}$ , and no exemptions are required as long as regulations are not too strict.

## 5 Impact on Price Levels, Risk–Taking, Depth and Volatility

The imposition of the VaR constraints affects the equilibria directly, with interesting results on risk-taking, liquidity, and volatility. We present our main results in a series of Propositions, with all proofs relegated to the Appendix.

### 5.1 Prices and Risk Premia

From equations (3) and (4) we know that the equilibrium pricing mapping is

$$Q(\boldsymbol{\epsilon}, \bar{v}) = \frac{1}{R_f} \left[ \hat{\boldsymbol{\mu}} - \Psi(\kappa(\boldsymbol{\epsilon}, \bar{v})) \hat{\boldsymbol{\Sigma}}(\boldsymbol{\theta}^a - \boldsymbol{\epsilon}) \right] \quad (5)$$

with  $\kappa(\boldsymbol{\epsilon}, \bar{v}) := \sqrt{\frac{(\boldsymbol{\theta}^a - \boldsymbol{\epsilon})' \hat{\boldsymbol{\Sigma}} (\boldsymbol{\theta}^a - \boldsymbol{\epsilon})}{\bar{v}}}$ . Since in economies where regulation is binding the reward-to-variability ratio is higher than in unregulated economies,  $\Psi > \gamma$ , it follows from (5) that at equilibrium, a binding risk-regulation induces lower prices for a risky asset  $j$  compared to the unconstrained economy iff the covariance of asset  $j$ 's payoff with the payoff of the residual market portfolio  $\boldsymbol{\theta}^a - \boldsymbol{\epsilon}$ , equivalently the beta, is positive,  $(\hat{\boldsymbol{\Sigma}})_{j^{\text{th row}}}(\boldsymbol{\theta}^a - \boldsymbol{\epsilon}) > 0$ , and higher prices otherwise. Therefore equity risk premia are higher the more tightly regulated the economy is:

**Proposition 2 (Equity Risk Premia)** *Let  $\bar{v}_2 < \bar{v}_1$ . Then  $\mu_i(\boldsymbol{\epsilon}, \bar{v}_2) - R_f > \mu_i(\boldsymbol{\epsilon}, \bar{v}_1) - R_f$ .*

where  $\mu_i(\epsilon, \bar{v}) := \hat{\mu}_i/Q_i(\epsilon, \bar{v})$  is the expected return on asset  $i$ . It is indeed easy to see that the CAPM with respect to the residual market portfolio holds. For instance, the excess return on asset  $i$  is  $\mu_i - R_f = \beta_{RM,i}(\mu_{RM} - R_f)$ , where in turn  $\mu_{RM} - R_f = \frac{\Psi \hat{\sigma}_{RM}^2}{q_{RM}}$ . The tighter the economy is regulated, the higher  $\Psi$  and the lower  $q_{RM}$ , generating higher expected excess returns.<sup>7</sup> Intuitively, a more tightly regulated economy transfers risk from the less risk-averse to the more risk-averse investors for markets to clear. But the latter need to be induced to buy into the risk by more advantageous prices, i.e. by higher expected returns.

Clearly, our static model is too simple to capture the complexity of the equity premium puzzle (as outlined by Mehra and Prescott, 1985; Weil, 1989), but many proposed solutions (see e.g. Constantinides, 1990; Epstein and Zin, 1990; Ferson and Constantinides, 1991; Benartzi and Thaler, 1995; Campbell and Cochrane, 1999; Barberis et al., 2001) involve modifying preferences in order to allow risk-appetite to play a larger role than in the time-separable expected utility base model. In that sense we outline one further channel that could be further exploited in a more general and explicitly dynamic version of this model. If the stylized coefficients of risk-aversion are too low to match asset returns when using frictionless models, maybe the additional degree of effective risk-aversion  $\Psi - \gamma$  due to risk-taking constraints, such as the ones imposed by the regulatory environment, may account for a fraction of the unexplained expected excess returns.

## 5.2 Risk–Taking

Consider the implications of the VaR constraint on those agents who are directly affected by the constraint and those only indirectly affected by the constraint. Denote by  $m$  the index of the marginal RFI whose risky holdings hit the regulatory maximum and by  $v_*(\epsilon)$  the weakest level of regulation for which all RFIs hit their VaR constraints,  $v_*(\epsilon) := \frac{(\theta^a - \epsilon)' \hat{\Sigma}(\theta^a - \epsilon)}{[\eta(1-\ell) + (1-\eta) \ln \ell^{-1}]^2}$ .

### Proposition 3 (Effects on Risk–Taking and Risk–Sharing)

- (i) *Less risk averse inframarginal RFIs,  $h \in [\ell, m)$ , have less risk appetite in the presence of VaR constraints,  $\alpha^h + \phi^{h,r} = m > \alpha^h$ , while the risk appetite of the more risk averse,  $h \in [m, 1]$ , remains unchanged at  $\alpha^h$ .*
- (ii) *The lower the admissible risk-taking level  $\bar{v}$ , the more RFIs hit their risk-taking constraints, i.e. the index of the marginal RFI,  $m$ , rises.*

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<sup>7</sup> $\hat{\sigma}_{RM}^2$  is the variance of the payoff of the residual market portfolio, and therefore exogenous. The price of the market portfolio,  $q_{RM} := \mathbf{q}'(\theta^a - \epsilon)$  is given by  $R_f^{-1} \hat{\mu}'(\theta^a - \epsilon) - R_f^{-1} \Psi(\theta^a - \epsilon)' \hat{\Sigma}(\theta^a - \epsilon)$ , decreasing unambiguously in  $\Psi$ .

(iii) *Those RFIs that are more risk-averse hold riskier portfolios in the presence of VaR constraints than they would otherwise. Risk-taking is therefore more uniform among RFIs in a regulated economy. For a given  $\epsilon$ , in the limit  $\bar{v} = v_*(\epsilon)$  and all constrained FIs hold the same risky portfolio.*

Item (i) is intuitive: considering the RFI's maximization program, we see that the effective risk-aversion of RFI  $h$  is  $\alpha^h + \phi^h$ . Now since the less risk-averse RFIs hold riskier portfolios, they hit the risk-constraint earlier than more risk-averse RFIs, with their risk-sharing capacity impeded. And once the VaR constraint for RFI  $h$  binds, its effective risk-aversion equals  $m$ , which is the same for all RFI's whose VaR constraint is binding.

Items (ii) and (iii) show that as regulations are tightened, more agents hit the allowed risk-limit. Since the aggregate risk is unaffected by regulations (recall that we are ignoring the possibility of a systemic collapse at this stage), a tightening of the regulatory risk constraint shifts risk from the less risk averse agents (the agents in the interval  $[m, m + \frac{\partial m}{\partial \bar{v}} d\bar{v}]$ ) via an appropriate price change to both the UFIs and to the more risk-averse regulated institutions. In other words, a binding regulatory risk constraint implies that regulated financial institutions effectively become more uniform in behavior since their attitudes to risk become more homogeneous. Risk-taking constraints force each regulated investor whose constraint binds to hold the same (maximal) amount of risk  $\bar{v}$ , no matter what his degree of risk-aversion  $\alpha^h \in [\ell, 1]$ . In the limit for a very tight policy,  $\bar{v} = v_*(\epsilon)$ , all RFIs hold the same risky portfolio.

If one does interpret RFIs close to  $\ell$  as banks (since they effectively take deposits and assume risks by investing in productive projects), then a stricter VaR regulation limits the natural role of these institutions, and optimal risk-sharing is compromised. This points out a potentially perverse implication of otherwise well-intentioned prudential regulations: by limiting the allowable level of risk that can be taken, at equilibrium, the risk-averse agents may be induced to hold portfolios that are riskier than the ones they would otherwise have held. Not only is risk-sharing shut down in the limit, the endogenous nature of prices induces risk-averse agents to take on *additional* risk.

### 5.3 Depth

The risk constraint affects the depth of the markets directly. In our context, *depth* (defined below) is an appropriate measure of liquidity, or alternatively the inverse depth, also called *shallowness*. The shallowness  $\mathcal{S}(\epsilon, \bar{v})$ , of the entire market is defined as the maximal extent to which an additional (unit-size) market order for a portfolio impacts its price. With this definition in mind, and with the formal definition and proofs in the Appendix, we can state:

**Proposition 4 (Depth)** *Depth (“liquidity”) is lower the tighter the constraint (i.e. the*

smaller  $\bar{v}$ ),  $\frac{\partial \mathcal{S}(\epsilon, \bar{v})}{\partial \bar{v}} < 0$  for all  $\epsilon \in \mathcal{E}$ . In particular, depth is lower in the regulated economy than in the unconstrained economy for any  $\epsilon \in \mathcal{E}$ .

Refer to Figure 3 for an illustration. No RFI's risk taking constraint is binding for  $\epsilon \in [\underline{\theta}^a(\bar{v}), \bar{\theta}^a(\bar{v})]$ . We have not made any assumptions regarding the distribution of  $\epsilon$ . However, in most cases we expect  $\epsilon < \theta^a$  because otherwise the value of noise traders' aggregate demand exceeds the value of all assets in the economy, i.e. in aggregate, the noise traders corner the market, in which case they really cease to be noise traders. In the Proposition that follows, we assume that  $\epsilon < \theta^a$  and that  $N = 1$ . This implies that the pricing function is concave over the relevant domain, and in most interesting cases (large negative noise trades, or restrictive regulations) the pricing function is strictly concave. The same can be shown for  $N > 1$  given the proper restrictions on the domain of noise trades.

**Proposition 5 (Bulls v.s. Bears)** *Assume  $N = 1$  and that regulations are sufficiently strict so that some agents are hitting the regulatory constraint at  $\epsilon = 0$ ,  $\bar{v} < \left(\frac{\sigma \theta^a}{\ell \ln \ell^{-1}}\right)^2$ . Also, assume that  $\mathbb{P}^\epsilon([\theta^a, \infty)) = 0$ .*

*Then, inflows raise prices less than outflows lower them.*

This widespread phenomenon has been dubbed by traders as “going up by the stairs and coming down by the elevator.”

## 5.4 Volatility

In the single asset case, inspection of Figure 3 reveals that the asset price becomes more volatile the stricter the VaR constraints are. In other words, uniform shallowness implies ex-ante volatility. The single-asset intuition can then be extended to the general case:

**Proposition 6 (Volatility)** *Assume that  $\mathbf{E} \subseteq \mathcal{E}(\bar{v})$ , that  $\bar{v}' > \bar{v}$  and that at least some agents face binding VaR constraints at  $\bar{v}$  on a non-null subset of  $\mathbf{E}$ . Equilibrium prices are more volatile in the economy with tighter regulations,  $\bar{v}$ , than in economy  $\bar{v}'$ . In particular, there is more volatility in the constrained economy than in the unconstrained economy.*

The basic intuition behind these results is that the less risk-averse RFIs (i.e.  $h$  close to  $\ell$ ), who are the first ones to hit the risk-taking constraint, have greater impact on pricing in the unconstrained economy. The trades which were previously absorbed by the more risk neutral RFIs, now have to be absorbed by the more risk averse. However, the more risk averse are less willing to absorb these (additional) market orders than the less risk averse. Hence the imposition of the risk constraint reduces market depth, and the market impact of a market order is larger. Since the arrival of market orders is random, this generates more volatile asset prices.

## 5.5 Comovements

It is a well-established fact that during periods of crises and instability, asset classes that would typically behave independently one from the other suddenly comove. This phenomenon is often referred to as “contagion.” One transmission mechanism frequently mentioned in the press is that margin calls on one asset class compel global investors to liquidate other assets, thereby repercuting a local shock through the entire financial system. Alternatively, in a theoretical model Kyle and Xiong (2001) attribute this fact to wealth effects and Kodres and Pritsker (2002) to what they call cross market rebalancing. But it has not been clear in theory why these comovements occur especially during crisis, and what the impact of risk-regulations on contagion could be. By the multi-asset nature of our model, we can naturally show the following:

**Proposition 7 (Comovements)** *Assets that are intrinsically statistically independent (i.e. the payoffs as well as the liquidity shocks of the assets considered are statistically mutually independent) become positively correlated due to risk-regulations.*

Even if two asset classes are payoff-independent and hit by independent noise trader shocks, if the regulations are strict enough to bind over a set of positive measure, then a large liquidity shock hitting one asset class will induce the VaR regulation to bind for some RFIs. These RFIs will subsequently need to adjust their global risk position, thereby creating comovements in asset prices among classes that would seem to be unrelated. Furthermore, these comovements would be detectable mostly in crisis situations, for during usual market conditions the VaR constraints do not bind. It would therefore seem that the VaR constraints bear one further seed of instability by not only creating asset price volatility, but by inducing correlations during the exact periods where such correlations are most dangerous.

## 6 Systemic Crises and Risk Regulations

Above, we side-stepped the issue of systemic crisis, the *raison d’être* for regulatory VaR constraints, and we followed Basak and Shapiro (2001) by imposing risk regulation upon a frictionless economy. But from a more normative point of view it is obvious that these regulations lead to a Pareto inferior allocation, and hence it would be desirable to explicitly model systemic failures. In specifying what constitutes a systemic crash, we follow a common definition, i.e. that the financial and credit system collapses and real promises become effectively worthless. Furthermore, we can capture the contributing factors to systemic risk by following views expressed by the regulators. In particular, Crockett (2000) has argued that regulations are necessary because unregulated markets lead to output losses due to financial instability. He says that upswings bear the seed for a crisis eventually leading to lost output since in upswings “too many resources” flow into risky investments

in a particularly unbalanced fashion. Similarly, Marshall (1998) lays out what he believes are the 5 main features that characterize a systemic crisis:

1. Systemic risk must originate in the process of financing, that is the capital needed by a firm is provided by investors outside the firm.
2. A systemic crisis involves contagion.
3. In a crisis, investors cut back the liquidity they are willing to provide to firms.
4. A systemic crisis involves substantial real costs, in terms of losses to economic output and/or reductions in economic efficiency. Crises must hurt Main Street, not just Wall Street.
5. A systemic crisis calls for a policy response.

It is a widely held belief that externalities and free-riding incentives (as in “too-big-to-fail”) imply that an excessive fraction of total risk is concentrated on a small but significant number of highly leveraged investors. In turn, it suffices that an unanticipated event transforms this imbalance into a systemic crisis.

## 6.1 Modelling Systemic Crisis

Unfortunately, we are not aware of any existing systemic crisis models that would exhibit those characteristics, or that could be straightforwardly integrated into our model. Refer to Allen and Gale (2000) and Diamond and Rajan (2001) for some of the known models attempting to capture systemic crises. We therefore set out to construct our own very stylized (so as to still generate closed-form solutions) modelling of the roots of systemic events. The main idea is to think of the shares as being rights to the output stream of firms. We now introduce an intermediate date at which firms may face a sudden (perfectly correlated across firms) liquidity need. This sudden demand for additional working capital conveys no information as to the worth of the firm, i.e. we abstract away from moral hazard issues and the like. In order to prevent total output loss, the existing shareholders are then asked to provide liquidity to the firms by lending them an amount of riskless assets proportional to their share holdings. We can view this stage either as pre-bankruptcy deliberations or as a stylized rights issue. This liquidity is reimbursed to them at date 1 with interest  $R_f$ , provided that the productive sector was able to raise sufficient liquidity. Refinancing may fail since the holders of large equity positions may not themselves have any or enough liquidity to lend to the firms. As outlined in the five defining features of a systemic crisis, financial contracting must be subjected to frictions in order to capture its essence. In this paper the frictions consist of the implicit assumptions that a) the productive sector must be refinanced as a whole (the outputs of the various firms are also inputs into

each other, say), and b) that markets are closed at the intermediate date. For instance, firms are not able to organize a syndicated bond issue quickly enough. The difficulty of issuing emergency bonds is well-known during periods of difficulty, not the least because a bond issue requires a bond rating, which can be a long and tedious process to get. Also, only a negligible fraction of firms in fact do have a rating.

The story about liquidity needs and systemic risk that one commonly hears (e.g. Marshall (1998)) is the one inspired from the Asian crisis. Complete details can be found in Corsetti et al. (1998) or Radelet and Sachs (1998) for instance. The debts of many East-Asian firms were dollar-denominated short-term borrowings, while investments were longer term. With the rapid appreciation of the dollar and the unwillingness of foreign lenders to roll over the debt (possibly due to a coordination problem), a liquidity event occurred. It was up to the mostly local stake-holders of the firms (and the governments and central banks) to provide the required dollar-denominated liquidity. Firms whose stake-holders had themselves over-extended on their own accounts failed.

While many authors attribute systemic fragility to an excessive piling-on of debt (e.g. Kindleberger (1978), Feldstein (1991)), those theories have relied explicitly or implicitly on irrationality. In our model no such irrationality is required to generate excessive leverage. Since each investor believes he is too small to affect the aggregate allocation, and therefore whether the refinancing is successful or not, he may therefore have an incentive to disregard the social cost of his actions and accumulate an excessively risky and leveraged position.<sup>8</sup> What is an “excessive” level for a FI is specified within the model, and depends on the actions of all other FIs.

Formally, assume that a liquidity event occurred, and that each shareholder is asked during the emergency meetings with the firms’ stakeholders to contribute  $K_i$  units of the riskless asset per unit of asset  $i$  held. This is similar to the fixed costs assumption in Marshall (1998). While shareholders do not have to come to the rescue of the productive sector by contributing working capital, it is a weakly dominant strategy to do so.<sup>9</sup> The total amount of riskless assets lent by  $(h, t)$  to the productive sector is therefore

$$L^{h,t} := \max\{0, \min\{y_0^{h,t}, \mathbf{K}'\mathbf{y}^{h,t}\}\}$$

and the financing shortfall stemming from investor  $(h, t)$  is (recall that  $\mathbf{K}'\mathbf{y}^{h,t}$  could be

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<sup>8</sup>Even if the FI had an impact and was aware of it, it would nevertheless overaccumulate risk since most of the benefits of holding liquidity accrue to society as a whole rather than to the FI in private.

<sup>9</sup>One could, as in Marshall (1998) (also see Radelet and Sachs (1998)), and at the expense of further complications, change the payoffs of the game slightly so as to allow for a coordination game. In that new game, if sufficiently many shareholders come to the rescue of the firms, then it is also in my interest to do so, while it is optimal for me not to contribute liquidity if insufficiently many other investors also refrain. As a further step, one could then introduce a coordinating device, such as the one used in global games (Carlsson and Van Damme (1993)), to derive a unique equilibrium for the refinancing game. The results of our paper as to the costs and benefits of VaR regulation would not however be qualitatively affected, but the interpretation may be closer to certain accounts of recent financial crises.



negative):

$$S^{h,t} := \max\{0, \mathbf{K}'\mathbf{y}^{h,t} - L^{h,t}\}$$

Aggregate shortfall is

$$S(\boldsymbol{\epsilon}, \bar{v}) := \eta \int_{\ell}^1 S^{h,r} dh + (1 - \eta) \int_{\ell}^1 S^{h,u} dh$$

Aggregate output  $\mathbf{d}'\boldsymbol{\theta}^a + R_f\theta_0^a$  collapses to zero if refinancing fails, i.e. if the proceeds are too low. The assumption that output completely collapses is made for simplicity only. For some given  $\bar{S}$  determined by the aggregate production technology,<sup>10</sup>

$$\{\text{Liquidity event occurs}\} \text{ and } \{S(\boldsymbol{\epsilon}, \bar{v}) > \bar{S}\} \Rightarrow \mathbf{d}'\boldsymbol{\theta}^a + R_f\theta_0^a = 0 \text{ a.s.}$$

The ex-ante probability of this happening is

$$\mathcal{P}(\bar{v}) := \mathbb{P}^{\epsilon}(\{\text{Liquidity event occurs}\} \cap \{S(\boldsymbol{\epsilon}, \bar{v}) > \bar{S}\})$$

In the second period one of two possible outcomes occurs: *normal market conditions* occur with endogenous probability  $1 - \mathcal{P}(\bar{v})$ , while a *systemic collapse* happens with probability  $\mathcal{P}(\bar{v})$ . The probability  $\mathcal{P}(\bar{v})$  depends on the chosen distribution of risk among the agents as discussed above. The RFI's ex-ante program (before  $\boldsymbol{\epsilon}$  is realized) consists in choosing demand schedules to solve

### Problem 1 (Risk-Constrained Ex-ante Problem)

$$\begin{aligned} & \max_{\{\mathbf{y}^h, y_0^h\}: \mathbb{R}^N \mapsto \mathbb{R}^{N+1}\}} \mathcal{P}(\bar{v})u^h(0) + (1 - \mathcal{P}(\bar{v}))E[u^h(x^h) | S(\boldsymbol{\epsilon}, \bar{v}) \leq \bar{S}] \\ \text{subject to} & \quad y_0^h + \mathbf{q}'\mathbf{y}^h \leq \theta_0^h + \mathbf{q}'\boldsymbol{\theta}^h \\ & \quad x^h = W^h \equiv \mathbf{d}'\mathbf{y}^h + R_f L^h + R_f(y_0^h - L^h) = \mathbf{d}'\mathbf{y}^h + R_f y_0^h \\ & \quad \mathbf{y}^{h'} \hat{\boldsymbol{\Sigma}} \mathbf{y}^h \leq \bar{v} \end{aligned}$$

Since individual institutions are negligible, this formulation gives rise to the free-riding externality mentioned above. Each financial institution chooses to neglect the effect of their actions on  $\mathcal{P}(\bar{v})$ . Given the manner by which we capture systemic risk, the RFI's Problem 1 is therefore equivalent, at each given  $\mathbf{q}$ , to the benchmark problem solved before, and the demand schedules and equilibrium prices derived there remain valid.

To make the regulatory problem interesting and transparent, we mention the following uniformity assumption:

**Assumption [U]**  $(\theta_0^{h,t}, \boldsymbol{\theta}^{h,t}) = \frac{1}{1-\ell}(\theta_0^a, \boldsymbol{\theta}^a)$ , all  $(h, t)$ .

<sup>10</sup> $\bar{S}$  could be modelled as a random variable. In a more explicitly dynamic model,  $\bar{S}$  would possibly depend upon the business cycle and other state-variables, such as past investments.

This assumption in particular excludes that the less risk-averse financial institutions have more wealth to manage. We believe this is uncontroversial, in particular in view of the fact that more free-wheeling funds are typically more highly leveraged.<sup>11</sup> If assumption [U] does not hold, then stricter VaR regulation can make systemic crises more likely. The reason is that regulation forces well capitalized risk tolerant institutions to sell risky assets to undercapitalized, but more risk averse, institutions. The latter may not have the required deep pockets to bail out the firms that face a sudden liquidity need, and the firms may fail. Proposition 8 implies that if [U] holds, the VaR regulations are effective in reducing the probability of a systemic crash, at least if a sufficiently large fraction of FIs are regulated:

**Proposition 8** *For a given  $\epsilon$ , consider the regulatory levels  $\bar{v} \in [v_*(\epsilon), \infty)$ . If  $\eta = 1$  and if [U] holds, the shortfall  $S$  is reduced as  $\bar{v}$  is lowered. It follows that the probability of a systemic event  $\mathcal{P}(\bar{v})$  is lowered as well. Furthermore, if  $\theta_0^a \geq \mathbf{K}'\theta^a$ , then  $\lim_{\bar{v} \rightarrow v_*+} S = 0$ .*

*When FIs are left out of the regulatory umbrella,  $\eta < 1$ , the effectiveness of the VaR regulations in preventing a systemic collapse may be weakened, and can even lead to a higher likelihood of a systemic collapse.*

If  $\eta = 1$ , the regulator faces a cost-benefit tradeoff. Imposing more stringent rules (a drop in  $\bar{v}$ ) induces suboptimal risk-sharing, more shallow asset markets and more volatile allocations during normal market conditions, possibly even leading to a complete market breakdown at time zero, but it also reduces the probability of a systemic failure,  $\text{Prob}(S(\epsilon, \bar{v}) > \bar{S})$ , which would have led to a loss of output and consumption. The reason this policy is effective is that the less risk-averse FIs hold large amounts of the risky portfolio if regulation is weak, and therefore will have to borrow at the riskless rate to finance such a risky holding, unless they are endowed with large amounts of assets to start with, which we exclude by condition [U]. Stricter risk-limits curb both the amount of risky assets held by the less risk-averse FIs as well as their required leverage, and therefore make it more likely that such institutions are able to take part in the refinancing of the firms. Recall that in the limit for  $\bar{v} = v_*$  all FIs hold the same risky portfolio. If all FIs are equally endowed, then all portfolios held are exactly the same. So if there is enough of the riskless asset in the economy to fully refinance firms to start with,  $\theta_0^a \geq \mathbf{K}'\theta^a$ , then each FI contributes the full amount required by the firms.

Perversely, if  $\eta < 1$  then a stricter VaR regulation prevents RFIs from taking on the risk, which is therefore shifted partly towards the more risk-averse RFIs and partly towards the UFIs, and in particular to the less risk-averse UFIs which may already hold levered portfolios of risky assets. The latter effect can dominate and lead, for a range of  $\eta$ , to an increase in the refinancing shortfalls and therefore put the financial viability of the system at risk.

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<sup>11</sup>We could have imposed weaker conditions at the expense of additional complications, such as **Assumption [U']**: For each  $\epsilon \in \mathbf{E}$ ,  $\mathbf{E}$  a compact neighbourhood of 0,  $\{\bar{S}, \{\theta^{h,t}, \theta_0^{h,t}\}_{h \in [\ell, 1], t \in \{r, u\}}\}$  satisfies  $S(\epsilon, \bar{v}) > \bar{S}$  for  $\bar{v}$  sufficiently low, and  $S(\epsilon, \bar{v}) \leq \bar{S}$  for  $\bar{v}$  sufficiently high.

This argument provides, in particular in light of the 1998 Russian crisis, a rationale of why regulators are pushing for more inclusive VaR regulations, and highlights the tradeoff regulators must balance between a *market breakdown*, defined as the situation where financial markets at time zero cannot clear ( $(\bar{v}, \epsilon)$  so that  $\bar{v} < v_*(\epsilon)$ ), and a *systemic collapse* in which the financial imbalances cause the collapse, at time 1, of the real productive sector, and (for simplicity) all output drops to zero ( $(\bar{v}, \epsilon)$  such that  $\bar{v} \geq v_*(\epsilon)$  and  $S(\epsilon; \bar{v}) > \bar{S}$ ).

## 7 Conclusion

The ultimate aim of financial risk regulations is the reduction of systemic risk. In general, this objective necessitates restricting financial institutions in their ability to assume risk. Even if this objective is laudable, in the real world specific policy instruments must be employed in the regulation of financial institutions.

We consider the existing VaR regulations in a two period general equilibrium model. Our results indicate that a naïve implementation of risk constraints may have unintended adverse consequences which would need to be balanced with the benefits of a more stable financial system.

A major contentious issue that has arisen with the VaR based market risk regulations is uniformity of application. It may be justified in the name of fairness (exempting hedge funds may penalize regulated institutions), as well as in the name of efficiency. Uniformity reduces the free-riding externalities, either arising from FIs ignoring the social cost of leverage, or from a moral hazard problem as in “too big to fail.” However this very uniformity of application also leads to some of the adverse effects of VaR regulations, not least in the form of allocational inefficiencies due to stifled risk-sharing. Our model could be viewed as one of the first formal models in which such questions can be asked and thought through, restrictive as it may be.

Among those adverse effects, the present regulations and the new proposals seem to insufficiently consider the fact that risk is endogenous. The lesson here is similar to the Lucas critique (Lucas, 1976). We demonstrate that regulating risk-taking changes the statistical properties of financial risk, rendering risk modelling all the more challenging. In particular, during crisis, VaR constraints change the risk appetite of financial institutions, effectively harmonizing their preferences. It is this effect which may be most damaging, since during crises it leads to higher correlations, higher volatility, larger drops in prices, worse risk-sharing and lower liquidity than would be realized in the absence of risk regulations.

Making regulation less universal (lowering  $\eta$ ), while alleviating the issues just raised and guaranteeing that asset markets can clear orderly, may lead to a higher probability of a systemic event. And worse, with UFIs, making regulations stricter may raise both the welfare costs linked to risk-sensitive regulation just mentioned *and* financial fragility, i.e. the likelihood of a systemic crisis. This suggests that a welfare maximizing policy calls for

intermediate levels of  $(\eta, \bar{v})$ .

## A Proofs

**Lemma 1 (Optimal Portfolio)** *The optimal portfolio of risky assets for RFI  $(h, t)$  has the mean-variance form*

$$\mathbf{y}^{h,t} = \frac{1}{\alpha^h + \phi^{h,t}} \hat{\Sigma}^{-1} (\hat{\boldsymbol{\mu}} - R_f \mathbf{q}) \quad (6)$$

where  $\phi^{h,u} := 0$  and  $\phi^{h,r} := \frac{2\lambda^{h,r}}{E^d[u^{h'}(W^h)]} \geq 0$ , with  $\lambda^{h,r}$  being the Lagrange multiplier of the VaR constraint. The effective degree of risk-aversion,  $\alpha^h + \phi^{h,r}$  is independent of the initial wealth  $W_0^h$  and only depends on  $\alpha^h$ ,  $\mathbf{q}$  and  $\bar{v}$ .

*Proof :*

The program consists in solving (the superscript  $d$  indicates that the expectation is computed with respect to the probability of the payoffs  $\mathbf{d}$ )

$$\max_{\{\mathbf{y}^h, y_0^h\}} E^d [u^h(d_0[\theta_0^h + \mathbf{q}'\boldsymbol{\theta}^h - \mathbf{q}'\mathbf{y}^h] + \mathbf{d}'\mathbf{y}^h)] - \lambda^h [\mathbf{y}^{h'} \hat{\Sigma} \mathbf{y}^h - \bar{v}]$$

The FOCs (the program is strictly convex, so the FOCs are both necessary and sufficient) are  $E^d [u^{h'}(W^h)(\mathbf{d} - d_0\mathbf{q})] = 2\lambda^h \hat{\Sigma} \mathbf{y}^h$ , or equivalently

$$\text{Cov}^d(u^{h'}(W^h), \mathbf{d}) + E^d [u^{h'}(W^h)] E[\mathbf{d}] - d_0 E^d [u^{h'}(W^h)] \mathbf{q} = 2\lambda^h \hat{\Sigma} \mathbf{y}^h$$

Next, by Stein's Lemma [recall that Stein's Lemma asserts that if  $x$  and  $y$  are multivariate normal, if  $g$  is everywhere differentiable and if  $E[g'(y)] < \infty$ , then  $\text{Cov}(x, g(y)) = E[g'(y)]\text{Cov}(x, y)$ ] and the fact that  $\text{Cov}^d(\mathbf{d}, W^h) = \text{Cov}^d(\mathbf{d}, \mathbf{d}'\mathbf{y}^h) = \hat{\Sigma} \mathbf{y}^h$  we get that:

$$\mathbf{y}^h = \frac{1}{\alpha^h + \phi^h} \hat{\Sigma}^{-1} [\hat{\boldsymbol{\mu}} - d_0 \mathbf{q}]$$

where we also used the fact that in this CARA–Normal setup  $\frac{-E^d[u^{h''}]}{E^d[u^{h'}]} = \alpha^h$ , and where we defined  $\phi^h := \frac{2\lambda^h}{E^d[u^{h'}]}$ .

Finally, we'll derive the expression for  $\alpha^h + \phi^h$  and show that it does not depend on the wealth of the institution. In order to accomplish this, we first need to find an expression for  $\phi^h$ . To simplify expressions, define

$$\boldsymbol{\rho} := (\hat{\boldsymbol{\mu}} - R_f \mathbf{q})' \hat{\Sigma}^{-1} (\hat{\boldsymbol{\mu}} - R_f \mathbf{q}) \quad (7)$$

It can easily be established that

$$\mathbf{y}^{h'} \hat{\Sigma} \mathbf{y}^h = \bar{v} \quad (\text{and } \lambda^h \geq 0) \Rightarrow \alpha^h + \phi^h = \sqrt{\frac{\boldsymbol{\rho}}{\bar{v}}} \quad (8)$$

$$\mathbf{y}^{h'} \hat{\Sigma} \mathbf{y}^h < \bar{v} \quad (\text{so } \lambda^h = 0) \Rightarrow \alpha^h + \phi^h = \alpha^h \quad (9)$$

Indeed, assume that  $\mathbf{y}^{h'} \hat{\Sigma} \mathbf{y}^h = \bar{v}$ . Since  $y^h = \frac{1}{\alpha^h + \phi^h} \hat{\Sigma}(\hat{\boldsymbol{\mu}} - R_f \mathbf{q})$ , this expression becomes  $\left(\frac{1}{\alpha^h + \phi^h}\right)^2 \rho = \bar{v}$ . Of course, if  $\mathbf{y}^{h'} \hat{\Sigma} \mathbf{y}^h < \bar{v}$  then  $\lambda^h = 0$  and thus  $\phi^h = 0$ .

This implies that  $\alpha^h + \phi^h$  is independent of  $W_0^h$  for given prices,

$$\alpha^h + \phi^h = \max \left\{ \alpha^h, \sqrt{\frac{\rho}{\bar{v}}} \right\} \quad (10)$$

Indeed, assume first that  $\mathbf{y}^{h'} \hat{\Sigma} \mathbf{y}^h < \bar{v}$ . Then by (9) we have that  $\alpha^h + \phi^h = \alpha^h$ , so we need to show that  $\alpha^h \geq \sqrt{\frac{\rho}{\bar{v}}}$ . Now since  $\mathbf{y}^{h'} \hat{\Sigma} \mathbf{y}^h = \alpha^{h-2} \rho$ , we know that  $\alpha^{h-2} \rho < \bar{v}$ , so that indeed  $\alpha^h > \sqrt{\frac{\rho}{\bar{v}}}$ . Next, assume that  $\mathbf{y}^{h'} \hat{\Sigma} \mathbf{y}^h = \bar{v}$ . Then from (8)  $\alpha^h + \phi^h = \sqrt{\frac{\rho}{\bar{v}}}$ . So we need to establish that  $\alpha^h \leq \sqrt{\frac{\rho}{\bar{v}}}$ , which follows from  $\phi^h \geq 0$ . ■

**Proof of Proposition 1** A **competitive equilibrium** is a price mapping  $Q$  together with its domain  $\mathcal{E}$ ,  $Q : \mathcal{E} \rightarrow \mathbb{R}^N$ , an asset allocation  $(h \in [\ell, 1], t \in \{r, u\}, \boldsymbol{\epsilon} \in \mathcal{E}) \mapsto (\mathbf{y}^{h,t}, y_0^{h,t})(\boldsymbol{\epsilon})$  and a consumption allocation  $(h \in [\ell, 1], t \in \{r, u\}, \boldsymbol{\epsilon} \in \mathcal{E}) \mapsto x^{h,t}(\boldsymbol{\epsilon})$  such that

- (i) Given any  $\boldsymbol{\epsilon} \in \mathcal{E}$  and  $\mathbf{q} \in Q(\boldsymbol{\epsilon})$ ,  $(\mathbf{y}^{h,t}, y_0^{h,t}, x^{h,t})$  solve FI  $(h, t)$ 's optimization problem, and this is true for all FIs  $(h, t) \in [\ell, 1] \times \{r, u\}$ .
- (ii) Markets for risky assets clear,  $\eta \int_{\ell}^1 \mathbf{y}^{h,r} dh + (1 - \eta) \int_{\ell}^1 \mathbf{y}^{h,u} dh + \boldsymbol{\epsilon} = \boldsymbol{\theta}^a$ , for each  $\boldsymbol{\epsilon} \in \mathcal{E}$ .

Before we proceed to the proof, notice that by Walras' Law, the markets for the riskless asset and for consumption clear if the market for risky assets clears. Indeed, denote aggregated FIs quantities by a superscript  $a$ : for any quantity  $x$ ,  $\int_{\ell}^1 (\eta x^{h,r} + (1 - \eta) x^{h,u}) dh = x^a$ . Walras' Law at times 0 and 1 says that  $(y_0^a - \theta_0^a + \epsilon_0) + \mathbf{q}'(\mathbf{y}^a - \boldsymbol{\theta}^a + \boldsymbol{\epsilon}) = 0$  [W0] and  $x^a + x^\epsilon = d_0(y_0^a + \theta_0^\epsilon + \epsilon_0) + \mathbf{d}'(\mathbf{y}^a + \boldsymbol{\theta}^\epsilon + \boldsymbol{\epsilon})$  [W1]. So assume that  $\mathbf{y}^a - \boldsymbol{\theta}^a + \boldsymbol{\epsilon} = 0$ . Then by [W0] the market for the riskless asset clears as well, and by [W1] we immediately have  $x^a + x^\epsilon = d_0(\theta_0^a + \theta_0^\epsilon) + \mathbf{d}'(\boldsymbol{\theta}^a + \boldsymbol{\theta}^\epsilon)$  under "normal market conditions."

We now exhibit a solution to the fixed-point problem of existence. Fix some  $\boldsymbol{\epsilon} \in \mathbf{E}$  and assume first that  $\ell > 0$ . Recall from (4) that

$$\begin{aligned} \Psi^{-1} &= \eta \int_{\ell}^1 \frac{1}{\alpha^h + \phi^{h,r}} dh + (1 - \eta) \int_{\ell}^1 \frac{1}{\alpha^h} dh \\ &= \eta \int_{I_1} \frac{1}{\alpha^h} dh + \eta \int_{I_2} \sqrt{\frac{\bar{v}}{\rho}} dh + (1 - \eta) \int_{\ell}^1 \frac{1}{\alpha^h} dh \end{aligned} \quad (11)$$

where  $I_1 := \{h \in [\ell, 1] : \alpha^h > \sqrt{\frac{\rho}{\bar{v}}}\}$  and  $I_2 := \{h \in [\ell, 1] : \alpha^h \leq \sqrt{\frac{\rho}{\bar{v}}}\}$ .

In order to solve for the equilibrium, we can either express  $\Psi$  (from (11)) as a function of  $\mathbf{q}$  and then solve (3) for  $\mathbf{q}$ , or we can use (3) to express  $\mathbf{q}$  as a function of  $\Psi$  and then solve (11) for  $\Psi$ . We chose the latter approach for obvious reasons.

For convenience, we establish some preliminary calculations and notation. First, insert the pricing relation (3) into the definition of  $\rho$  from (7) to get the expression  $\sqrt{\rho} = \Psi \sqrt{(\boldsymbol{\theta}^a - \boldsymbol{\epsilon})' \hat{\Sigma} (\boldsymbol{\theta}^a - \boldsymbol{\epsilon})}$ . Second, define the relation  $\kappa(\boldsymbol{\epsilon}) := \sqrt{\frac{(\boldsymbol{\theta}^a - \boldsymbol{\epsilon})' \hat{\Sigma} (\boldsymbol{\theta}^a - \boldsymbol{\epsilon})}{\bar{v}}} \equiv \Psi^{-1} \sqrt{\frac{\bar{p}}{\bar{v}}}$ .  $\kappa(\boldsymbol{\epsilon})$  represents the ratio of the standard deviation of the dividends of the residual market portfolio  $\boldsymbol{\theta}^a - \boldsymbol{\epsilon}$  to the maximal allowable standard deviation of the payoffs of individual portfolios. By our assumption that  $\alpha^h = h$ , we can then define the ranges  $I_1 = \{h \in [\ell, 1] : \alpha^h = h > \Psi\kappa(\boldsymbol{\epsilon})\}$  and  $I_2 = \{h \in [\ell, 1] : \alpha^h = h \leq \Psi\kappa(\boldsymbol{\epsilon})\}$  to get the functional equation

$$\Psi^{-1} = \eta \int_{I_1} h^{-1} dh + \eta |I_2| (\Psi\kappa(\boldsymbol{\epsilon}))^{-1} + (1 - \eta) \int_{\ell}^1 h^{-1} dh \quad (12)$$

For simplicity, we drop the explicit dependence of  $\kappa$  upon  $\boldsymbol{\epsilon}$  wherever no confusion arises. We have to distinguish 3 cases:

$$\int_{I_1} \frac{1}{h} dh = \begin{cases} \int_{\Psi\kappa}^1 \frac{1}{h} dh = -\ln(\Psi\kappa) & ; \Psi\kappa \in [\ell, 1] \\ \int_1^1 \frac{1}{h} dh = 0 & ; \Psi\kappa > 1 \\ \int_{\ell}^1 \frac{1}{h} dh = -\ln(\ell) & ; \Psi\kappa < \ell \end{cases} ; \quad I_2 = \begin{cases} [\ell, \Psi\kappa] & ; \Psi\kappa \in [\ell, 1] \\ [\ell, 1] & ; \Psi\kappa > 1 \\ \emptyset & ; \Psi\kappa < \ell \end{cases}$$

The equilibrium relations thus become

$$\Psi^{-1} = \begin{cases} -\eta \ln(\ell) + (1 - \eta) \ln \ell^{-1} & ; \Psi\kappa < \ell \\ \eta [-\ln(\Psi\kappa) + (\Psi\kappa - \ell)(\Psi\kappa)^{-1}] + (1 - \eta) \ln \ell^{-1} & ; \Psi\kappa \in [\ell, 1] \\ \eta(1 - \ell)(\Psi\kappa)^{-1} + (1 - \eta) \ln \ell^{-1} & ; \Psi\kappa > 1 \end{cases}$$

By tedious manipulations it can be shown (details available from the authors) that there is a unique  $\Psi$  solving this system. Since  $\boldsymbol{\epsilon}$  affects  $\Psi$  only in as far as it affects  $\kappa$ , it is useful to point out that the mapping  $\kappa \mapsto \Psi(\kappa; \eta, \ell)$  (we often drop the dependency on  $\eta$  and  $\ell$  if no ambiguity arises and write the mapping as  $\Psi(\kappa)$ ), can be characterized as follows: if  $\eta < 1$  and  $\ell \in (0, 1)$ , then

$$\Psi(\kappa) = \begin{cases} \frac{1}{\ln \ell^{-1}} & ; \kappa \in [0, \ell \ln \ell^{-1}] \\ \frac{-\kappa - \eta \ell}{\eta \kappa W_{-1}(-(\kappa \eta^{-1} + \ell) \exp(-\ln \ell^{-1} + \eta^{-1} \ln \ell))} & ; \kappa \in (\ell \ln \ell^{-1}, \eta(1 - \ell) + (1 - \eta) \ln \ell^{-1}) \\ \frac{1 - (1 - \ell) \kappa^{-1} \eta}{(1 - \eta) \ln \ell^{-1}} & ; \kappa \geq \eta(1 - \ell) + (1 - \eta) \ln \ell^{-1} \end{cases} \quad (13)$$

where  $W_{-1}(\cdot)$  is the non-principal (lower) branch of the Lambert  $W$ -correspondence. Recall that the Lambert  $W$ -correspondence is defined as the multivariate inverse of the function  $w \mapsto we^w$ . Notice that the mapping  $\Psi$  is continuous and that by construction the equilibrium  $\Psi$  satisfies  $\Psi \geq \gamma$ .

If  $\eta = 1$ , then

$$\Psi(\kappa) = \begin{cases} \frac{1}{\ln \ell^{-1}} & ; \kappa \in [0, \ell \ln \ell^{-1}] \\ -\frac{\kappa + \ell}{\kappa W_{-1}(-(\kappa + \ell)e^{-1})} & ; \kappa \in (\ell \ln \ell^{-1}, 1 - \ell) \\ \text{any number} \geq \frac{1}{1 - \ell} & ; \kappa = 1 - \ell \\ \text{undefined} & ; \kappa > 1 - \ell \end{cases}$$

Over the entire domain the correspondence  $\Psi(\kappa)$  is illustrated in figure (2). In the case for  $\eta = 1$  and  $\kappa = 1 - \ell$ , which is equivalent to  $\epsilon$  being on the boundary of  $\mathcal{E}$ , the equilibrium can be shown to exhibit real indeterminacy. ■

**Proof of Proposition 3** Most results follow from the properties of the index of the marginal regulated investor,  $m := \Psi\kappa$ . Define  $v_*(\epsilon)$  as the weakest level of regulation for which all RFIs hit their VaR constraints,

$$v_*(\epsilon) := \frac{(\theta^a - \epsilon)' \hat{\Sigma} (\theta^a - \epsilon)}{[\eta(1 - \ell) + (1 - \eta) \ln \ell^{-1}]^2}$$

If  $\eta = 1$ ,  $v_*(\epsilon)$  is also the strictest level of regulation, given  $\epsilon$ , that can still be supported by an equilibrium. Similarly, define  $v^*(\epsilon)$  as the weakest level of regulation for which there is an agent whose risk-taking constraint is binding,

$$v^*(\epsilon) := \begin{cases} \frac{(\theta^a - \epsilon)' \hat{\Sigma} (\theta^a - \epsilon)}{(\ell \ln \ell^{-1})^2} & \text{if } \eta > 0 \\ 0 & \text{if } \eta = 0 \end{cases}$$



Indeed, this can be established as follows. Almost no regulated investor's constraint is binding iff  $v^h \leq \bar{v}$  all  $h$ , i.e. iff  $\left(\frac{1}{\alpha^h + \phi^{h,r}}\right)^2 \Psi^2 \kappa^2 \bar{v} \leq \bar{v}$  ( $\forall h$ ), iff  $\left(\frac{1}{\alpha^h + \phi^{h,r}}\right)^2 \Psi^2 (\boldsymbol{\theta}^a - \boldsymbol{\epsilon})' \hat{\Sigma} (\boldsymbol{\theta} - \boldsymbol{\epsilon}) \leq \bar{v}$  iff  $\left(\frac{\frac{1}{\alpha^h}}{\int_{\ell}^1 \frac{1}{\alpha^k} dk}\right)^2 (\boldsymbol{\theta}^a - \boldsymbol{\epsilon})' \hat{\Sigma} (\boldsymbol{\theta}^a - \boldsymbol{\epsilon}) \leq \bar{v}$  for all  $h$ . Now this holds for all  $h$  iff it holds for  $h = \ell$ , and plugging in  $\frac{1}{\int_{\ell}^1 \frac{1}{\alpha^k} dk} = [\ell \ln \ell^{-1}]^{-1}$  we get the stated result.

We summarize some properties of  $\Psi \kappa$  first. They are proved for  $\eta < 1$ , but they hold for  $\eta = 1$  as well as long as the parameters are such that  $\bar{v} \geq v_*(\boldsymbol{\epsilon})$ , the necessary and sufficient condition for an equilibrium to exist.

L0 For  $\ell > 0$  and  $\eta < 1$ ,  $\Psi(\kappa)$  is differentiable on the entire  $\mathbb{R}_+$  (follows from the implicit function theorem).

$$\frac{\partial \Psi}{\partial \kappa} = \begin{cases} 0 & ; \kappa \in (0, \ell \ln \ell^{-1}] \\ -\frac{\Psi(\kappa \eta \Psi - \eta \ell)}{\kappa^2 (\eta \Psi - (1 + \kappa^{-1} \ell \eta))} & ; \kappa \in (\ell \ln \ell^{-1}, \eta(1 - \ell) + (1 - \eta) \ln \ell^{-1}) \\ \frac{\eta(1 - \ell)}{\kappa^2 (1 - \eta) \ln \ell^{-1}} & ; \kappa \geq \eta(1 - \ell) + (1 - \eta) \ln \ell^{-1} \end{cases}$$

So  $\frac{\partial \Psi}{\partial \kappa} \geq 0$ , and  $> 0$  if  $\eta \in (0, 1)$  and  $\kappa > \ell \ln \ell^{-1}$ .

L1

$$\frac{\partial \Psi}{\partial \bar{v}} = \frac{\partial \Psi}{\partial \kappa} \frac{\partial \kappa}{\partial \bar{v}} \leq 0, \quad < 0 \quad \text{if } \boldsymbol{\theta}^a \neq \boldsymbol{\epsilon}, \eta > 0 \text{ and } \kappa > \ell \ln \ell^{-1}$$

L2  $\Psi \kappa$  is differentiable in  $\bar{v}$ .  $\frac{\partial \Psi \kappa}{\partial \bar{v}} \leq 0, < 0$  iff  $\boldsymbol{\theta}^a \neq \boldsymbol{\epsilon}$ , in particular for  $\bar{v} \in (v_*(\boldsymbol{\epsilon}), v^*(\boldsymbol{\epsilon}))$ .

Indeed, for the index of the marginal RFI  $\Psi \kappa$ ,  $\frac{\partial \Psi \kappa}{\partial \bar{v}} = \left[ \Psi + \kappa \frac{\partial \Psi}{\partial \kappa} \right] \frac{\partial \kappa}{\partial \bar{v}} \leq 0, < 0$  iff  $\boldsymbol{\theta}^a \neq \boldsymbol{\epsilon}$  (since  $\frac{\partial \kappa}{\partial \bar{v}} = 0$  iff  $\boldsymbol{\theta}^a = \boldsymbol{\epsilon}$ ). Notice that if  $\boldsymbol{\theta}^a = \boldsymbol{\epsilon}$ , then the interval  $(v_*(\boldsymbol{\epsilon}), v^*(\boldsymbol{\epsilon}))$  is empty.

L3  $\Psi \kappa = 1$  at  $\bar{v} = v_*(\boldsymbol{\epsilon})$ .

Indeed,  $\lim_{\kappa \rightarrow \eta(1 - \ell) + (1 - \eta) \ln \ell^{-1}} \kappa \Psi(\kappa) = \lim_{\kappa \rightarrow \eta(1 - \ell) + (1 - \eta) \ln \ell^{-1}} \frac{\kappa - (1 - \ell) \eta}{(1 - \eta) \ln \ell^{-1}} = 1$

L4  $\Psi\kappa = \ell$  at  $\bar{v} = v^*(\epsilon)$ .

Indeed,  $\lim_{\kappa \rightarrow \ell \ln \ell^{-1}} \kappa \Psi(\kappa) = \lim_{\kappa \rightarrow \ell \ln \ell^{-1}} \frac{\kappa}{\ln \ell^{-1}} = \ell$ .

The proof of (i) is obvious, and (ii) is simply (L2). As to (iii), denote the payoff variance of investor  $h$  by  $v^h := \mathbf{y}^{h'} \hat{\Sigma} \mathbf{y}^h = \frac{1}{(\alpha^h + \phi^h)^2} \Psi^2 \bar{v} \kappa^2 = \frac{1}{(\alpha^h + \phi^h)^2} \Psi^2 (\boldsymbol{\theta}^a - \epsilon)' \hat{\Sigma} (\boldsymbol{\theta}^a - \epsilon)$ . We need to show that  $\frac{\partial v^h}{\partial \bar{v}} < 0$  if  $h > \Psi\kappa$ . So assuming that  $h > \Psi\kappa$  (so that in particular  $\Psi\kappa < 1$ ), it is indeed easy to see that

$$\begin{aligned} \frac{\partial v^h}{\partial \bar{v}} &= \frac{2}{h^2} (\boldsymbol{\theta}^a - \epsilon)' \hat{\Sigma} (\boldsymbol{\theta}^a - \epsilon) \Psi \frac{\partial \Psi}{\partial \bar{v}} \\ &\leq 0; \quad < 0 \text{ if } \boldsymbol{\theta}^a \neq \epsilon, \eta > 0 \text{ and } \kappa > \ell \ln \ell^{-1} \end{aligned}$$

Taken together with (ii), this shows that as regulations are tightened marginally, the risk is taken away from the agents in the interval  $[\Psi\kappa, \Psi\kappa + \frac{\partial \Psi\kappa}{\partial \bar{v}} d\bar{v}]$  and transferred (via an appropriate price change) to the unregulated and to the regulated but more risk-averse.

For a given  $\epsilon$ , let us lower  $\bar{v}$  towards  $v_*(\epsilon)$ . Observations L1 to L4 taken together say that as  $\bar{v}$  becomes smaller, the marginal RFI tends to 1, in which case all investors' risk-taking constraint is binding, and thus all hold the same risky portfolio because each RFI's effective risk-aversion is identical in that situation. ■

**Proof of Proposition 4** Shallowness is formally defined as

$$\mathcal{S}(\epsilon, \bar{v}) := \max_{\boldsymbol{\theta} \text{ subject to } \|\boldsymbol{\theta}\|=1} |\boldsymbol{\theta}' dQ| = \max_{\boldsymbol{\theta} \text{ subject to } \|\boldsymbol{\theta}\|=1} |\boldsymbol{\theta}' (\partial_\epsilon Q) \boldsymbol{\theta}|$$

As preliminaries, let us record the following useful results:

J1  $\frac{\partial \kappa}{\partial \bar{v}} = -\frac{1}{2} \frac{\kappa}{\bar{v}}$ , from the definition of  $\kappa$ , and  $\partial_\epsilon \kappa = \kappa^{-1} \bar{v}^{-1} \hat{\Sigma} (\epsilon - \boldsymbol{\theta}^a)$ .

J2  $\partial_{\epsilon, \bar{v}}^2 \Psi = -\frac{1}{2\bar{v}} \left[ \kappa \frac{\partial^2 \Psi}{\partial \kappa^2} + \frac{\partial \Psi}{\partial \kappa} \right] \partial_{\epsilon} \kappa$ . Indeed, since  $\partial_{\bar{v}} \Psi = \frac{\partial \Psi}{\partial \kappa} \frac{\partial \kappa}{\partial \bar{v}}$ , we know from J1 that  $\partial_{\epsilon, \bar{v}}^2 \Psi = \frac{d}{d\epsilon} \left( \frac{\partial \Psi}{\partial \kappa} \frac{\partial \kappa}{\partial \bar{v}} \right) = \frac{\partial \kappa}{\partial \bar{v}} \frac{\partial^2 \Psi}{\partial \kappa^2} \partial_{\epsilon} \kappa + \frac{\partial \Psi}{\partial \kappa} \partial_{\epsilon} \left( -\frac{1}{2} \frac{\kappa}{\bar{v}} \right)$ .

J3  $\partial_{\epsilon} Q$  is positive definite (downward-sloping equilibrium inverse demand). Indeed,  $\partial_{\epsilon} Q = R_f^{-1} \left[ \Psi \hat{\Sigma} - \hat{\Sigma}(\theta^a - \epsilon)(\partial_{\epsilon} \Psi)' \right] = R_f^{-1} \left[ \Psi \hat{\Sigma} + \hat{\Sigma}(\theta^a - \epsilon) (\theta^a - \epsilon)' \hat{\Sigma} \kappa^{-1} \bar{v}^{-1} \frac{\partial \Psi}{\partial \kappa} \right]$ , positive definite.

The idea of the proof is to show that  $\partial_{\epsilon, \bar{v}}^2 Q$  is negative definite. Intuitively, we want to show that the market impact of a trade goes up as the regulation is tightened, i.e. that  $\frac{\partial}{\partial \bar{v}} |(d\mathbf{q})'(d\epsilon)| = \frac{\partial}{\partial \bar{v}} [(d\mathbf{q})'(d\epsilon)] < 0$  since  $(d\mathbf{q})'(d\epsilon) > 0$  as  $\partial_{\epsilon} Q$  is positive definite by J3. Now this expression equals  $\frac{\partial}{\partial \bar{v}} [(d\epsilon)' \partial_{\epsilon} Q(d\epsilon)] = (d\epsilon)' \partial_{\epsilon, \bar{v}}^2 Q(d\epsilon) < 0$  for all  $d\epsilon \neq 0$ , but that's the definition of negative definiteness.

Before we show that  $\partial_{\epsilon, \bar{v}}^2 Q(d\epsilon)$  is negative definite, we want to relate this idea with the definition of shallowness,  $\mathcal{S}(\epsilon, \bar{v}) := \max_{\theta} |\theta'(\partial_{\epsilon} Q)\theta|$  such that  $\|\theta\| = 1$ , namely that  $\frac{\partial \mathcal{S}}{\partial \bar{v}} < 0$  iff  $\partial_{\epsilon, \bar{v}}^2 Q$  negative definite. Indeed, pick any  $\theta$  such that  $\|\theta\| = 1$ , then it is immediate that  $\frac{\partial(\theta' \partial_{\epsilon} Q \theta)}{\partial \bar{v}} = \theta' (-\partial_{\epsilon, \bar{v}}^2 Q) \theta$ , which proves the claim. In some sense, a tighter  $\bar{v}$  makes  $\partial_{\epsilon} Q$  “more positive definite.”

The pricing function is  $Q(\epsilon, \bar{v}) = R_f^{-1} \left[ \hat{\mu} - \Psi \hat{\Sigma}(\theta^a - \epsilon) \right]$ , from which we can deduce that  $\partial_{\bar{v}} Q = -R_f^{-1} \hat{\Sigma}(\theta^a - \epsilon) \frac{d\Psi}{d\bar{v}}$ , and furthermore that  $\partial_{\epsilon, \bar{v}}^2 Q = R_f^{-1} \hat{\Sigma} \frac{d\Psi}{d\bar{v}} - R_f^{-1} \hat{\Sigma}(\theta^a - \epsilon) \partial_{\epsilon, \bar{v}}^2 \Psi$ .

This expression can be simplified, using J2, to

$$\partial_{\epsilon, \bar{v}}^2 Q = -\frac{1}{2} R_f^{-1} \frac{\partial \Psi}{\partial \kappa} \frac{\kappa}{\bar{v}} \hat{\Sigma} - \frac{1}{2} R_f^{-1} \bar{v}^{-2} \left[ \frac{\partial^2 \Psi}{\partial \kappa^2} \kappa + \frac{\partial \Psi}{\partial \kappa} \right] \left[ \hat{\Sigma}(\theta^a - \epsilon)(\theta^a - \epsilon)' \hat{\Sigma} \right] \kappa^{-1}$$

The first term is negative definite, while the second one is negative semidefinite. Indeed, it can be shown that the expression  $\left[ \frac{\partial^2 \Psi}{\partial \kappa^2} \kappa + \frac{\partial \Psi}{\partial \kappa} \right]$  is strictly positive, while the term  $\left[ \hat{\Sigma}(\theta^a - \epsilon)(\theta^a - \epsilon)' \hat{\Sigma} \right]$  is clearly positive semidefinite. This concludes the proof that  $\partial_{\epsilon, \bar{v}}^2 Q$  is negative definite. ■

**Proof of Proposition 5** We would like to establish Figure 3. Assume that  $N = 1$ , so that there is a single risky asset. We can deduce the following three characteristics of the pricing function, for  $\bar{v}' > \bar{v}$ :

(i) We have

$$\begin{aligned} Q(\epsilon, \bar{v}) &< Q(\epsilon, \bar{v}') && \text{on } \epsilon < \theta^a - \frac{\sqrt{\bar{v}}}{\sigma} \ell \ln \ell^{-1} \\ Q(\epsilon, \bar{v}) &= Q(\epsilon, \bar{v}') && \text{on } \epsilon \in \left[ \theta^a - \frac{\sqrt{\bar{v}}}{\sigma} \ell \ln \ell^{-1}, \theta^a + \frac{\sqrt{\bar{v}}}{\sigma} \ell \ln \ell^{-1} \right] \\ Q(\epsilon, \bar{v}) &> Q(\epsilon, \bar{v}') && \text{on } \epsilon > \theta^a + \frac{\sqrt{\bar{v}}}{\sigma} \ell \ln \ell^{-1} \end{aligned}$$

(ii)  $\frac{\partial Q}{\partial \epsilon} > 0$ , and

(iii)  $\frac{\partial Q}{\partial \epsilon}(\bar{v}) > \frac{\partial Q}{\partial \epsilon}(\bar{v}')$ .

(iv) The graph is symmetrical with center  $(\theta^a, Q(\theta^a, \infty))$ .

We briefly prove (i) here. First, assume that  $\epsilon < \theta^a - \frac{\sqrt{\bar{v}}}{\sigma} \ell \ln \ell^{-1}$ . Then  $\kappa = \frac{\sigma|\theta^a - \epsilon|}{\sqrt{\bar{v}}} = \frac{\sigma(\theta^a - \epsilon)}{\sqrt{\bar{v}}} > \ell \ln \ell^{-1}$ . It follows that  $\Psi > \gamma$  and  $\frac{\partial \Psi}{\partial \kappa} > 0$ , so that  $\Psi(\epsilon, \bar{v}) > \Psi(\epsilon, \bar{v}')$  since  $\bar{v}' > \bar{v}$  implies that  $\kappa(\epsilon, \bar{v}') < \kappa(\epsilon, \bar{v})$ . Since  $Q(\epsilon, \bar{v}) = R_f^{-1} [\bar{\mu} - \Psi \sigma^2 (\theta^a - \epsilon)]$ , this results in  $Q(\epsilon, \bar{v}) < Q(\epsilon, \bar{v}')$  for all  $\epsilon < \theta^a - \frac{\sqrt{\bar{v}}}{\sigma} \ell \ln \ell^{-1}$ .

Next, assume that  $\epsilon \in \left[ \theta^a - \frac{\sqrt{\bar{v}}}{\sigma} \ell \ln \ell^{-1}, \theta^a + \frac{\sqrt{\bar{v}}}{\sigma} \ell \ln \ell^{-1} \right]$ , so that  $\theta^a - \epsilon \in \left[ -\frac{\sqrt{\bar{v}}}{\sigma} \ell \ln \ell^{-1}, +\frac{\sqrt{\bar{v}}}{\sigma} \ell \ln \ell^{-1} \right]$ , which implies that  $\frac{\sigma(\theta^a - \epsilon)}{\sqrt{\bar{v}}} \in [-\ell \ln \ell^{-1}, \ell \ln \ell^{-1}]$  and finally that  $\kappa = \frac{\sigma|\theta^a - \epsilon|}{\sqrt{\bar{v}}} \in [0, \ell \ln \ell^{-1}]$ . Then  $\Psi = \gamma$ , and  $Q(\epsilon, \bar{v}) = Q(\epsilon, +\infty)$ , and in particular that  $Q(\epsilon, \bar{v}) = Q(\epsilon, \bar{v}')$ .

This completes the proof of item (i).

In order to prove (iv), we fix any  $\bar{v}$  and we need to verify  $Q(\theta^a + \eta) - Q(\theta^a) = Q(\theta^a) - Q(\theta^a - \eta)$ ,  $\eta > 0$ . This equation boils down to  $\Psi(\kappa(\theta^a + \eta)) = \Psi(\kappa(\theta^a - \eta))$ . But  $\kappa(\theta^a + \eta) = \frac{\sigma|\eta|}{\sqrt{\bar{v}}} = \kappa(\theta^a - \eta)$  completes the proof of (iv).

Taken together, the equilibrium correspondence looks as on figure 3. ■

**Proof of Proposition 6** Consider some asset  $i \in \{1, \dots, N\}$ . As noted in earlier proofs, we know that  $\partial_\epsilon Q(\epsilon; \bar{v})$  is positive definite, and that  $\partial_{\epsilon, \bar{v}}^2 Q(\epsilon; \bar{v})$  is negative definite. From there we can deduce that for  $\bar{v}' > \bar{v}$ ,  $\frac{\partial Q_i}{\partial \epsilon_i}(\epsilon; \bar{v}) \geq \frac{\partial Q_i}{\partial \epsilon_i}(\epsilon; \bar{v}')$ , and with strict inequality if the VaR is binding somewhere in the economy, which we assume from now on. More precisely, we assume, in order to make the problem interesting, that  $E$  is such that the VaR constraint binds for at least some agents at the stricter level of regulation  $\bar{v}$  over a subset of  $E$ .

Now define the random variables  $F$  and  $G$  as  $F := f(\epsilon) := Q_i(\epsilon; \bar{v})$  and similarly  $G := g(\epsilon) := Q_i(\epsilon; \bar{v}')$ . For a given realization of  $\epsilon_{\setminus i}$ , we know that  $\partial f / \partial \epsilon_i > \partial g / \partial \epsilon_i > 0$ , and therefore that  $H := h(\epsilon) := f(\epsilon) - g(\epsilon)$  satisfies  $\partial h / \partial \epsilon_i > 0$ . Since  $\text{Var}(F) = \text{Var}(G) + \text{Var}(H) + 2\text{Cov}(H, G)$ , all we need to show is that  $\text{Cov}(H, G) > 0$ . Since  $h$  and  $g$  are both one-to-one in  $\epsilon_i$  for a given  $\epsilon_{\setminus i}$ , there is an increasing differentiable function  $h_g$  satisfying  $H = h_g(G; \epsilon_{\setminus i})$ . Notice that by the mean-value theorem, using the notation  $\bar{G} := E[g(\epsilon) | \epsilon_{\setminus i}]$ , there is an intermediate point  $G^*$  such that:

$$h_g(G; \epsilon_{\setminus i}) = h_g(\bar{G}; \epsilon_{\setminus i}) + h'_g(G^*(G; \epsilon_{\setminus i}); \epsilon_{\setminus i})(G - \bar{G})$$

Now, using the Law of Iterated Expectations,

$$\begin{aligned} \text{Cov}(G, H) &= E[H(G - \bar{G})] \\ &= E_{\epsilon_{\setminus i}} [E_{\epsilon_i} [H(G - \bar{G}) | \epsilon_{\setminus i}]] \\ &= E_{\epsilon_{\setminus i}} [E_{\epsilon_i} [(h_g(\bar{G}; \epsilon_{\setminus i}) + h'_g(G^*(G; \epsilon_{\setminus i}); \epsilon_{\setminus i})(G - \bar{G})) (G - \bar{G}) | \epsilon_{\setminus i}]] \\ &= E_{\epsilon_{\setminus i}} \underbrace{[E_{\epsilon_i} [h'_g(G^*(G; \epsilon_{\setminus i}); \epsilon_{\setminus i})(G - \bar{G})^2 | \epsilon_{\setminus i}]]}_{\geq 0} > 0 \end{aligned}$$

■

**Proof of Proposition 7** Consider any two assets, say assets 1 and 2. Intrinsic independence requires  $\hat{\Sigma}$  diagonal,  $\epsilon_1 \perp \epsilon_2$ , and the absence of regulations so that  $\Psi = \varphi$ . Then  $Q_1(\epsilon_1)$  and  $Q_2(\epsilon_2)$ , so  $Q_1 \perp Q_2$ .

Define  $\mathbf{z} := \boldsymbol{\theta}^a - \boldsymbol{\epsilon}$ . Then, since  $Q_i = \hat{\mu}_i - \Psi(\boldsymbol{\epsilon})\hat{\boldsymbol{\Sigma}}_i(\boldsymbol{\theta}^a - \boldsymbol{\epsilon})$ ,

$$\text{Cov}(Q_1, Q_2) = \hat{\boldsymbol{\Sigma}}_{11}\hat{\boldsymbol{\Sigma}}_{22}\text{Cov}(\Psi z_1, \Psi z_2)$$

with  $\Psi(z_1, z_2)$ . Now fix  $z_1$ . Given  $z_1$ , both  $\Psi z_2$  and  $\Psi z_1$  are increasing monotonically in  $z_2$ , and therefore there exists, for a given  $z_1$ , a monotonically increasing function  $f$  such that

$$\Psi z_1 = f(\Psi z_2; z_1)$$

In other words,  $Q_1$  and  $Q_2$  are conditionally comonotonic. The Law of Iterated Expectations says

$$E[\Psi z_1(\Psi z_2 - E[\Psi z_2])] = E_{z_1} [E_{z_2}[f(\Psi z_2; z_1)(\Psi z_2 - E[\Psi z_2])|z_1]]$$

But from the proof of Proposition 6 we know that  $E_{z_2}[f(\Psi z_2; z_1)(\Psi z_2 - E[\Psi z_2])|z_1] > 0$ . ■

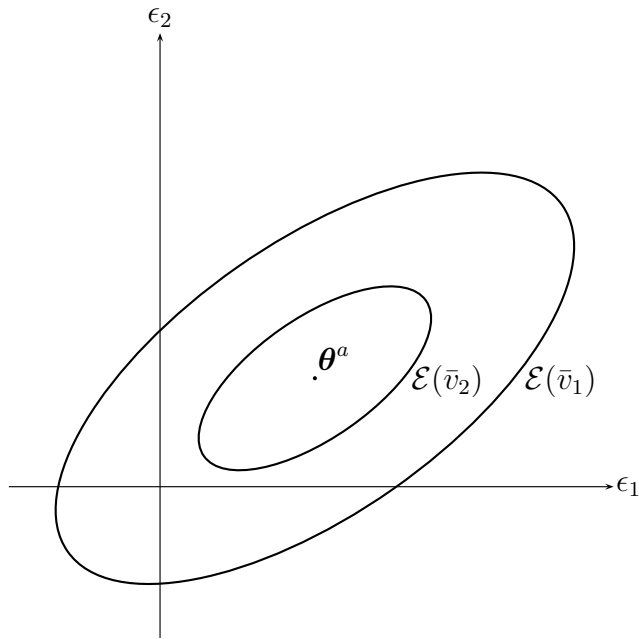


Figure 1: EQUILIBRIUM ELLIPSOIDS WITH INCREASINGLY RESTRICTIVE RISK CONSTRAINTS

In this scenario there are two assets, and in the absence of any regulations, equilibria exist for  $\epsilon \in \mathbb{R}^2$ . When the risk constraint is  $\bar{v}_1$ , the set of  $\epsilon$  that can be supported by an equilibrium is the larger ellipsoid, and includes zero noise trader demand. However a more restrictive constraint  $\bar{v}_2$  does not include zero net demand, and hence equilibria do not exist if noise trades are zero.

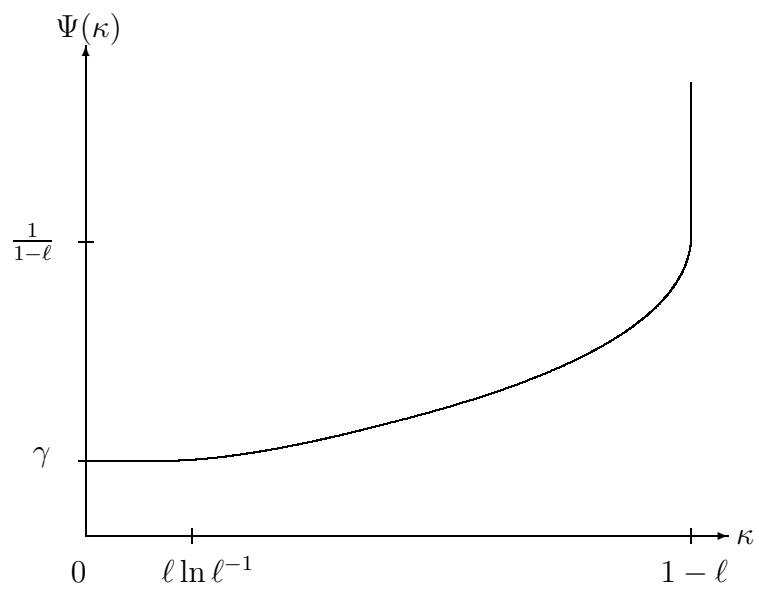


Figure 2: ILLUSTRATION OF THE REWARD-TO-RISK FUNCTION  $\Psi(\kappa)$  WHEN  $\eta = 1$  AND  $\ell > 0$ .



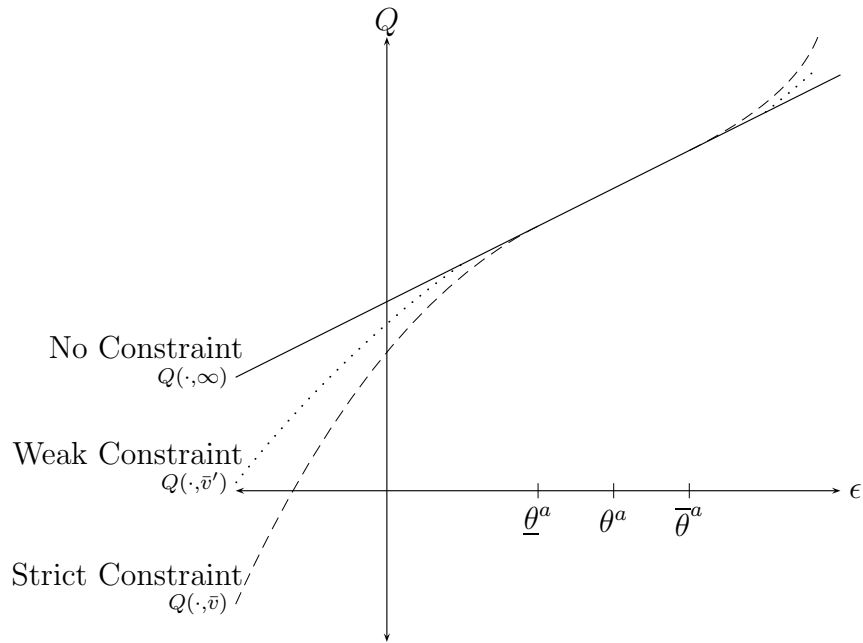


Figure 3: PRICING FUNCTION

The pricing function without constraints and with increasingly binding constraints,  $\infty > \bar{v}' > \bar{v}$ . The downside effects become more pronounced as the constraint becomes stricter.

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